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Casimir densities for two spherical branes in Rindler-like spacetimes

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Abstract: Wightman function, the vacuum expectation values of the field square and the energy-momentum tensor are evaluated for a scalar field obeying the Robin boundary conditions on two spherical branes in $(D+1)$-dimensional Rindler-like spacetime $Ri \times S^{D-1}$, with a two-dimensional Rindler spacetime Ri. This spacetime approximates the near horizon geometry of $(D+1)$-dimensional black hole. By using the generalized Abel-Plana formula, the vacuum expectation values are presented as the sum of single brane and second brane induced parts. Various limiting cases are studied. The vacuum forces acting on the branes are decomposed into the self-action and interaction terms. The interaction forces are investigated as functions of the brane locations and coefficients in the boundary conditions.

Keywords: Field Theories in Higher Dimensions, Large Extra Dimensions, Black Holes.

Contents

1. Introduction

Motivated by string/M theory, the AdS/CFT correspondence, and the hierarchy problem of particle physics, braneworld models were studied actively in recent years (for a review see [1]). In these models, our world is represented by a sub-manifold, a three-brane, embedded in a higher dimensional spacetime. In particular, a well studied example is when the bulk is an AdS space. The braneworld corresponds to a manifold with boundaries and all fields which propagate in the bulk will give Casimir-type contributions to the vacuum energy, and as a result to the vacuum forces acting on the branes. In dependence of the type of a field and boundary conditions imposed, these forces can either stabilize or destabilize the braneworld. In addition, the Casimir energy gives a contribution to both the brane and bulk cosmological constants and, hence, has to be taken into account in the self-consistent formulation of the braneworld dynamics. Motivated by these, the role of quantum effects in braneworld scenarios has received a great deal of attention. For a conformally coupled scalar this effect was initially studied in ref. [2] in the context of M-theory, and subsequently in refs. [3] for a background Randall-Sundrum geometry. The models with dS and AdS branes, and higher dimensional brane models are considered as well [4].

In view of these recent developments, it seems interesting to generalize the study of quantum effects to other types of bulk spacetimes. In particular, it is of interest to consider non-Poincaré invariant braneworlds, both to better understand the mechanism of localized gravity and for possible cosmological applications. Bulk geometries generated by higher-dimensional black holes are of special interest. In these models, the tension and the position of the brane are tuned in terms of black hole mass and cosmological constant and brane gravity trapping occurs in just the same way as in the Randall-Sundrum model. Braneworlds in the background of the AdS black hole were studied in [5]. Like pure

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AdS space the AdS black hole may be superstring vacuum. It is of interest to note that the phase transitions which may be interpreted as confinement-deconfinement transition in AdS/CFT setup may occur between pure AdS and AdS black hole [6]. Though, in the generic black hole background the investigation of brane-induced quantum effects is technically complicated, the exact analytical results can be obtained in the near horizon and large mass limit when the brane is close to the black hole horizon. In this limit the black hole geometry may be approximated by the Rindler-like manifold (for some investigations of quantum effects on background of Rindler-like spacetimes see [7] and references therein).

In the previous papers [8, 9] we have considered the vacuum densities induced by a spherical brane in the bulk $Ri \times S^{D-1}$, where Ri is a two-dimensional Rindler spacetime. Continuing in this direction, in the present paper we investigate the Wightman function, the vacuum expectation values of the field square and the energy-momentum tensor for a scalar field with an arbitrary curvature coupling parameter for two spherical branes on the same bulk. Though the corresponding operators are local, due to the global nature of the vacuum, these expectation values describe the global properties of the bulk and carry an important information about the physical structure of the quantum field at a given point. The expectation value of the energy-momentum tensor acts as the source of gravity in the Einstein equations and, hence, plays an important role in modelling a self-consistent dynamics involving the gravitational field. In addition to applications in braneworld models on the AdS black hole bulk, the problem under consideration is also of separate interest as an example with gravitational and boundary-induced polarizations of the vacuum, where all calculations can be performed in a closed form. Note that the vacuum densities induced by a single and two parallel flat branes in the bulk geometry $Ri \times R^{D-1}$ for both scalar and electromagnetic fields are investigated in [10, 11].

The paper is organized as follows. In the next section we consider the positive frequency Wightman functions in the region between two branes. On the basis of the generalized Abel-Plana formula, we present this function in the form of the sum of single brane and second brane induced parts. By using expression for the Wightman function, in section 3 we investigate the vacuum expectation values of the field square and the energy-momentum tensor. Various limiting cases of the general formulae are studied. In section 4 the vacuum forces acting on the branes due to the presence of the second brane are evaluated by making use of the expression for the radial vacuum stress. The main results of the paper are summarized in section 5.

2. Wightman function

We consider a scalar field $\varphi(x)$ propagating on background of $(D+1)$-dimensional Rindler-like spacetime $Ri \times S^{D-1}$. The corresponding metric is described by the line element

$$ds^2 = \xi^2 d\tau^2 - d\xi^2 - r_H^2 d\Sigma_{D-1}^2, \tag{2.1}$$

with the Rindler-like (τ, ξ) part and $d\Sigma_{D-1}^2$ is the line element for the space with positive constant curvature with the Ricci scalar $R = (D-2)(D-1)/r_H^2$. Line element (2.1) describes the near horizon geometry of $(D+1)$-dimensional topological black hole with the line element [12]

$$ds^2 = A_H(r)dt^2 - \frac{dr^2}{A_H(r)} - r^2 d\Sigma_{D-1}^2, \tag{2.2}$$

where $A_H(r) = k + r^2/l^2 - r_0^D/l^2 r^n$, $n = D - 2$, and the parameter k classifies the horizon topology, with $k = 0, -1, 1$ corresponding to flat, hyperbolic, and elliptic horizons, respectively. The parameter l is related to the bulk cosmological constant and the parameter r_0 depends on the mass of the black hole. In the non extremal case the function $A_H(r)$ has a simple zero at $r = r_H$, and in the near horizon limit, introducing new coordinates τ and ρ in accordance with

$$\tau = A_H'(r_H)t/2, \quad r - r_H = A_H'(r_H)\xi^2/4, \tag{2.3}$$

the line element is written in the form (2.1). Note that for a $(D + 1)$-dimensional Schwarzschild black hole [13] one has $A_H(r) = 1 - (r_H/r)^n$ and, hence, $A_H'(r_H) = n/r_H$.

The field equation is in the form

$$\left(g^{ik}\nabla_i\nabla_k + m^2 + \zeta R\right)\varphi(x) = 0, \tag{2.4}$$

where ζ is the curvature coupling parameter. Below we will assume that the field satisfies the Robin boundary conditions on the hypersurfaces $\xi = a$ and $\xi = b$, $a < b$,

$$\left(A_j + B_j\frac{\partial}{\partial\xi}\right)\varphi\bigg|_{\xi=j} = 0, \quad j = a, b, \tag{2.5}$$

with constant coefficients A_j and B_j. The Dirichlet and Neumann boundary conditions are obtained as special cases. In accordance with (2.3), the hypersurface $\xi = j$ corresponds to the spherical shell near the black hole horizon with the radius $r_j = r_H + A_H'(r_H)j^2/4$.

The branes divide the bulk into three regions corresponding to $0 < \xi < a$, $a < \xi < b$, and $b < \xi < \infty$. In general, the coefficients in the boundary conditions (2.5) can be different for separate regions. In the corresponding braneworld scenario based on the orbifolded version of the model the region between the branes is employed only and the ratio A_j/B_j for untwisted bulk scalars is related to the brane mass parameters c_j of the field by the formula [8]

$$\frac{A_j}{B_j} = \frac{1}{2}\left(c_j - \frac{\zeta}{j}\right), \; j = a, b. \tag{2.6}$$

For a twisted scalar the Dirichlet boundary conditions are obtained on both branes.

To evaluate the vacuum expectation values (VEVs) of the field square and the energy-momentum tensor we need a complete set of eigenfunctions satisfying the boundary conditions (2.5). In accordance with the problem symmetry, below we shall use the hyperspherical angular coordinates $(\vartheta, \phi) = (\theta_1, \theta_2, \ldots, \theta_n, \phi)$ on S^{D-1} with $0 \leqslant \theta_k \leqslant \pi$, $k = 1, \ldots, n$, and $0 \leqslant \phi \leqslant 2\pi$. In these coordinates the eigenfunctions in the region between the branes can be written in the form

$$\varphi_\alpha(x) = C_\alpha Z_{i\omega}^{(b)}(\lambda_l\xi, \lambda_l b)Y(m_k; \vartheta, \phi)e^{-i\omega\tau}, \tag{2.7}$$

where $m_k = (m_0 \equiv l, m_1, \ldots m_n)$, and $m_1, m_2, \ldots m_n$ are integers such that $0 \leqslant m_{n-1} \leqslant \cdots \leqslant m_1 \leqslant l$, $-m_{n-1} \leqslant m_n \leqslant m_{n-1}$. In eq. (2.7) $Y(m_k; \vartheta, \phi)$ is the spherical harmonic of degree l [14], and

$$Z_{i\omega}^{(j)}(u,v) = \bar{I}_{i\omega}^{(j)}(v) K_{i\omega}(u) - \bar{K}_{i\omega}^{(j)}(v) I_{i\omega}(u), \ j = a, b, \tag{2.8}$$

with $I_{i\omega}(x)$ and $K_{i\omega}(x)$ being the modified Bessel functions with the imaginary order,

$$\lambda_l = \frac{1}{r_H} \sqrt{l(l+n) + \zeta n(n+1) + m^2 r_H^2} \,. \tag{2.9}$$

Here and below for a given function $f(z)$ we use the barred notations

$$\bar{f}^{(j)}(z) = A_j f(z) + \frac{B_j}{j} z f'(z), \quad j = a, b. \tag{2.10}$$

Functions (2.7) satisfy the boundary condition on the brane $\xi = b$. From the boundary condition on the brane $\xi = a$ we find that the possible values for ω are roots to the equation

$$Z_{i\omega}(\lambda_l a, \lambda_l b) = 0, \tag{2.11}$$

with the notation

$$Z_\omega(u,v) = \bar{I}_\omega^{(b)}(v) \bar{K}_\omega^{(a)}(u) - \bar{K}_\omega^{(b)}(v) \bar{I}_\omega^{(a)}(u). \tag{2.12}$$

For a fixed λ_l, the equation (2.11) has an infinite set of real solutions with respect to ω. We will denote them by $\omega_n = \omega_n(\lambda_l a, \lambda_l b)$, $\omega_n > 0$, $n = 1, 2, \ldots$, and will assume that they are arranged in the ascending order $\omega_n < \omega_{n+1}$. In addition to the real zeros, in dependence of the values of the ratios jA_j/B_j, equation (2.11) can have a finite set of purely imaginary solutions. The presence of such solutions leads to the modes with an imaginary frequency and, hence, to the unstable vacuum. In the consideration below we will assume the values of the coefficients in boundary conditions (2.5) for which the imaginary solutions are absent and the vacuum is stable.

The coefficient C_α in (2.7) can be found from the normalization condition

$$r_H^{D-1} \int d\Omega \int_a^b \frac{d\xi}{\xi} \varphi_\alpha \overleftrightarrow{\partial}_\tau \varphi_{\alpha'}^* = i\delta_{\alpha\alpha'}. \tag{2.13}$$

where the integration goes over the region between two spheres. Substituting eigenfunctions (2.7) and using the relation $\int |Y(m_k; \vartheta, \phi)|^2 \, d\Omega = N(m_k)$ for spherical harmonics, one finds

$$C_\alpha^2 = \left. \frac{r_H^{1-D} \bar{I}_{i\omega}^{(a)}(\lambda_l a)}{N(m_k) \bar{I}_{i\omega}^{(b)}(\lambda_l b) \frac{\partial}{\partial\omega} Z_{i\omega}(\lambda_l a, \lambda_l b)} \right|_{\omega=\omega_n} . \tag{2.14}$$

The explicit form for $N(m_k)$ is given in [14] and will not be necessary for the following considerations in this paper.

First of all we evaluate the positive frequency Wightman function

$$G^+(x,x') = \langle 0 | \varphi(x) \varphi(x') | 0 \rangle, \tag{2.15}$$

where $|0\rangle$ is the amplitude for the corresponding vacuum state. The VEVs for the field square and the energy-momentum tensor are obtained from this function in the coincidence limit of the arguments. In addition, the Wightman function determines the response of a particle detector in given state of motion. By expanding the field operator over eigenfunctions and using the commutation relations one can see that

$$G^+(x,x') = \sum_\alpha \varphi_\alpha(x)\varphi_\alpha^*(x'). \tag{2.16}$$

Substituting eigenfunctions (2.7) into this mode sum formula and by making use of the addition theorem

$$\sum_{m_k} \frac{Y(m_k;\vartheta,\phi)}{N(m_k)} Y(m_k;\vartheta',\phi') = \frac{2l+n}{nS_D} C_l^{n/2}(\cos\theta), \tag{2.17}$$

for the Wightman function in the region between the branes one finds

$$\begin{aligned} G^+(x,x') = {} & \frac{r_H^{1-D}}{nS_D}\sum_{l=0}^\infty (2l+n)C_l^{n/2}(\cos\theta)\sum_{n=1}^\infty \frac{\bar{I}_{i\omega}^{(a)}(\lambda_l a)e^{-i\omega(\tau-\tau')}}{\bar{I}_{i\omega}^{(b)}(\lambda_l b)\frac{\partial}{\partial\omega}Z_{i\omega}(\lambda_l a,\lambda_l b)} \\ & \times Z_{i\omega}^{(b)}(\lambda_l\xi,\lambda_l b)Z_{i\omega}^{(b)}(\lambda_l\xi',\lambda_l b)\Big|_{\omega=\omega_n}. \end{aligned} \tag{2.18}$$

In this formula, $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is the volume of the unit $(D-1)$-sphere, $C_l^{n/2}(x)$ is the Gegenbauer polynomial of degree l and order $n/2$, θ is the angle between directions (ϑ,ϕ) and (ϑ',ϕ').

As the normal modes ω_n are not explicitly known and the terms with large n are highly oscillatory, the Wightman function in the form (sum over n the summation formula derived in ref. [11] on the basis of the generalized Abel-Plana formula [15]:

$$\sum_{n=1}^\infty \frac{\bar{I}_{-i\omega_n}^{(b)}(v)\bar{I}_{i\omega_n}^{(a)}(u)}{\frac{\partial}{\partial z}Z_{iz}(u,v)|_{z=\omega_n}}F(\omega_n) = \int_0^\infty dz\frac{\sinh\pi z}{\pi^2}F(z) - \int_0^\infty dz\frac{F(iz)+F(-iz)}{2\pi Z_z(u,v)}\bar{I}_z^{(a)}(u)\bar{I}_{-z}^{(b)}(v). \tag{2.19}$$

As a function $F(z)$ in this formula we choose

$$F(z) = \frac{Z_{iz}^{(b)}(\lambda_l\xi,\lambda_l\lambda b)Z_{iz}^{(b)}(\lambda_l\xi',\lambda_l b)}{\bar{I}_{iz}^{(b)}(\lambda_l b)\bar{I}_{-iz}^{(b)}(\lambda_l b)}e^{-iz(\tau-\tau')}. \tag{2.20}$$

The conditions for the formula (2.19) to be valid are satisfied if $a^2 e^{|\tau-\tau'|} < \xi\xi'$. For the Wightman function one obtains the expression

$$\begin{aligned} G^+(x,x') = {} & G^+(x,x';b) - \frac{r_H^{1-D}}{\pi nS_D}\sum_{l=0}^\infty (2l+n)C_l^{n/2}(\cos\theta)\int_0^\infty d\omega\,\Omega_{b\omega}(\lambda_l a,\lambda_l b) \\ & \times Z_\omega^{(b)}(\lambda_l\xi,\lambda_l b)Z_\omega^{(b)}(\lambda_l\xi',\lambda_l b)\cosh[\omega(\tau-\tau')], \end{aligned} \tag{2.21}$$

where

$$\Omega_{b\omega}(u,v) = \frac{\bar{I}_\omega^{(a)}(u)}{\bar{I}_\omega^{(b)}(v)Z_\omega(u,v)}. \tag{2.22}$$

In eq. (2.21)

$$G^+(x,x';b) = \frac{r_H^{1-D}}{\pi^2 nS_D}\sum_{l=0}^{\infty}(2l+n)C_l^{n/2}(\cos\theta)\int_0^\infty d\omega\sinh(\pi\omega)$$
$$\times e^{-i\omega(\tau-\tau')}\frac{Z_{i\omega}^{(b)}(\lambda_l\xi,\lambda b)Z_{i\omega}^{(b)}(\lambda_l\xi',\lambda_l b)}{\bar{I}_{i\omega}^{(b)}(\lambda_l b)\bar{I}_{-i\omega}^{(b)}(\lambda_l b)}, \tag{2.23}$$

is the Wightman function in the region $\xi < b$ for a single brane at $\xi = b$ and the second term on the right is induced by the presence of the brane at $\xi = a$. The function (2.21) is investigated in ref. [8] and can be presented in the form

$$G^+(x,x';b) = G_0^+(x,x') + \langle\varphi(x)\varphi(x')\rangle^{(b)}, \tag{2.24}$$

where $G_0^+(x,x')$ is the Wightman function for the geometry without boundaries and the part

$$\langle\varphi(x)\varphi(x')\rangle^{(b)} = -\frac{r_H^{1-D}}{\pi nS_D}\sum_{l=0}^{\infty}(2l+n)C_l^{n/2}(\cos\theta)\int_0^\infty d\omega\frac{\bar{K}_\omega^{(b)}(\lambda_l b)}{\bar{I}_\omega^{(b)}(\lambda_l b)}$$
$$\times I_\omega(\lambda_l\xi)I_\omega(\lambda_l\xi')\cosh[\omega(\tau-\tau')] \tag{2.25}$$

is induced in the region $\xi < b$ by the presence of the brane at $\xi = b$. Note that the representation (2.24) with (2.25) is valid under the assumption $\xi\xi' < b^2 e^{|\tau-\tau'|}$. As it has been shown in [8], the Wightman function for the boundary-free geometry may be written in the form

$$G_0^+(x,x') = \tilde{G}_0^+(x,x') - \frac{r_H^{1-D}}{\pi^2 nS_D}\sum_{l=0}^{\infty}(2l+n)C_l^{n/2}(\cos\theta)$$
$$\times\int_0^\infty d\omega e^{-\omega\pi}\cos[\omega(\tau-\tau')]K_{i\omega}(\lambda_l\xi)K_{i\omega}(\lambda_l\xi'), \tag{2.26}$$

where $\tilde{G}_0^+(x,x')$ is the Wightman function for the bulk geometry $R^2\times S^{D-1}$. Outside the horizon the divergences in the coincidence limit of the expression on the right of (2.26) are contained in the first term.

It can be seen that the Wightman function in the region between the branes can be also presented in the form

$$G^+(x,x') = G^+(x,x';a) - \frac{r_H^{1-D}}{\pi nS_D}\sum_{l=0}^{\infty}(2l+n)C_l^{n/2}(\cos\theta)\int_0^\infty d\omega\,\Omega_{a\omega}(\lambda_l a,\lambda_l b)$$
$$\times Z_\omega^{(a)}(\lambda_l\xi,\lambda_l a)Z_\omega^{(a)}(\lambda_l\xi',\lambda_l a)\cosh[\omega(\tau-\tau')], \tag{2.27}$$

with the notation

$$\Omega_{a\omega}(u,v) = \frac{\bar{K}_\omega^{(b)}(v)}{\bar{K}_\omega^{(a)}(u)Z_\omega(u,v)}. \tag{2.28}$$

In this representation,

$$G^+(x,x';a) = G_0^+(x,x') + \langle\varphi(x)\varphi(x')\rangle^{(a)} \tag{2.29}$$

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is the Wightman function in the region $\xi > a$ for a single brane at $\xi = a$, and

$$\langle\varphi(x)\varphi(x')\rangle^{(a)} = -\frac{r_H^{1-D}}{\pi n S_D}\sum_{l=0}^{\infty}(2l+n)C_l^{n/2}(\cos\theta)\int_0^{\infty}d\omega\frac{\bar{I}_\omega^{(a)}(\lambda_l a)}{\bar{K}_\omega^{(a)}(\lambda_l a)} \times K_\omega(\lambda_l\xi)K_\omega(\lambda_l\xi')\cosh[\omega(\tau-\tau')]. \tag{2.30}$$

Two representations of the Wightman function, given by eqs. (2.21) and (2.27), are obtained from each other by the replacements

$$a \rightleftarrows b, \quad I_\omega \rightleftarrows K_\omega. \tag{2.31}$$

In the coincidence limit the second term on the right of formula (2.21) is finite on the brane $\xi = b$ and diverges on the brane at $\xi = a$, whereas the second term on the right of eq. (2.27) is finite on the brane $\xi = a$ and is divergent for $\xi = b$. Consequently, the forms (2.21) and (2.27) are convenient for the investigations of the VEVs near the branes $\xi = b$ and $\xi = a$, respectively.

We have investigated the Whightman function in the region between two branes for an arbitrary ratio of boundary coefficients A_j/B_j. Note that in the orbifolded version of the model the integration in the normalization integral goes over two copies of the bulk manifold. This leads to the additional coefficient 1/2 in the expression (2.14) for the normalization coefficient C_α. Hence, the Whightman function in the orbifolded braneworld case is given by formula (2.21) with an additional factor 1/2 in the second term on the right and in formula (2.23). As it has been mentioned above this function corresponds to the braneworld in the AdS black hole bulk in the limit when the branes are close to the black hole horizon.

3. Casimir densities

3.1 VEV for the field square

In this section we will consider the VEVs of the field square and the energy-momentum tensor in the region between the branes. In the coincidence limit, taking into account the relation $C_l^{n/2}(1) = \Gamma(l+n)/\Gamma(n)l!$, from the formulae for the Wightman function one obtains two equivalent forms for the VEV of the field square:

$$\langle 0|\varphi^2|0\rangle = \langle 0_0|\varphi^2|0_0\rangle + \langle\varphi^2\rangle^{(j)} - \frac{r_H^{1-D}}{\pi S_D}\sum_{l=0}^{\infty}D_l\int_0^{\infty}d\omega\,\Omega_{j\omega}(\lambda_l a,\lambda_l b)Z_\omega^{(j)2}(\lambda_l\xi,\lambda_l j), \tag{3.1}$$

corresponding to $j = a$ and $j = b$, and $|0_0\rangle$ is the amplitude for the vacuum without boundaries,

$$D_l = (2l+D-2)\frac{\Gamma(l+D-2)}{\Gamma(D-1)l!} \tag{3.2}$$

is the degeneracy of each angular mode with given l. The VEV $\langle 0_0|\varphi^2|0_0\rangle$ is obtained from the corresponding Wightman function given by (2.26). For the points outside the horizon, the renormalization procedure is needed for the first term on the right only, which corresponds to the VEV in the geometry $R^2 \times S^{D-1}$. This procedure is realized in [8] on the base of the zeta function technique.

In eq. (3.1), the part $\langle\varphi^2\rangle^{(j)}$ is induced by a single brane at $\xi = j$ when the second brane is absent. For the geometry of a single brane at $\xi = a$, from (2.30) one has

$$\langle\varphi^2\rangle^{(a)} = -\frac{r_H^{1-D}}{\pi S_D}\sum_{l=0}^{\infty} D_l \int_0^{\infty} d\omega \frac{\bar{I}_\omega^{(a)}(\lambda_l a)}{\bar{K}_\omega^{(a)}(\lambda_l a)} K_\omega^2(\lambda_l \xi). \tag{3.3}$$

The expression for $\langle\varphi^2\rangle^{(b)}$ is obtained from (induced by the presence of the second brane. It is finite on the brane at $\xi = j$ and diverges for the points on the other brane. By taking into account the relation $Z_\omega^{(j)}(u,u) = B_j/j$, we see that for the Dirichlet boundary condition this term vanishes on the brane $\xi = j$.

Let us consider the behavior of the single brane part (3.3) in asymptotic regions of the parameters. In the limit $\xi \to a$ this part diverges and, hence, for points near the brane the main contribution comes from large values l. By making use of the corresponding uniform asymptotic expansions for the modified Bessel functions, to the leading order we find

$$\langle\varphi^2\rangle^{(a)} \approx -\frac{\delta_{B_a}\Gamma\left(\frac{D-1}{2}\right)}{(4\pi)^{\frac{D+1}{2}}(\xi - a)^{D-1}}, \tag{3.4}$$

where $\delta_{B_a} = 1$ for $B_a = 0$ and $\delta_{B_a} = -1$ for $B_a \neq 0$. Hence, near the brane the brane-induced part is negative for the Dirichlet boundary condition and is positive for non-Dirichlet boundary condition. At large distances from the brane, $\xi \gg r_H$, the dominant contribution into (3.3) comes from the $l = 0$ term and in the leading order we have

$$\langle\varphi^2\rangle^{(a)} \approx -\frac{r_H^{1-D}e^{-2\lambda_0\xi}}{2S_D\lambda_0\xi}\int_0^{\infty} d\omega \frac{\bar{I}_\omega^{(a)}(\lambda_0 a)}{\bar{K}_\omega^{(a)}(\lambda_0 a)}. \tag{3.5}$$

In the limit when the position of the brane tends to the horizon, $a \to 0$, with fixed ξ, we use the formulae for the modified Bessel functions with small values of the argument. The main contribution into the integral comes from the lower limit of the integration and we obtain the formula

$$\langle\varphi^2\rangle^{(a)} \approx -\frac{r_H^{1-D}\delta_{B_a}}{2\pi S_D \ln^2(r_H/a)}\sum_{l=0}^{\infty} D_l K_0^2(\lambda_l \xi). \tag{3.6}$$

In the limit $r_H \to 0$ the curvature for the background spacetime is large. In this limit λ_l is also large. The exception is the term $l = 0$ for a minimally coupled scalar field for which $\lambda_0 = m$. For large values λ_l the main contribution into the integral in (integration variable $x = \omega/\lambda_l a$, we estimate the integral by the Laplace method. This leads to the following result

$$\langle\varphi^2\rangle^{(a)} \approx -\frac{\delta_{B_a} r_H^{1-D} a}{4\sqrt{\pi}S_D\xi}\frac{\exp[-2\sqrt{\zeta n(n+1)}(\xi - a)/r_H]}{\sqrt{\lambda_0(\xi - a)}}. \tag{3.7}$$

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For a minimally coupled scalar field the contribution of the terms with $l \geqslant 1$ is suppressed by the factor $e^{-2\lambda_l(\xi-a)}$ and the dominant contribution comes from the $l=0$ term:

$$\langle\varphi^2\rangle^{(a)} = -\frac{r_H^{1-D}}{\pi S_D}\int_0^\infty d\omega \frac{\bar{I}_\omega^{(a)}(ma)}{\bar{K}_\omega^{(a)}(ma)} K_\omega^2(m\xi). \tag{3.8}$$

As we see, the behavior of the VEV in the high curvature regime is essentially different for minimally and non-minimally coupled fields.

In the near-horizon limit, $a, \xi \ll r_H$, the main contribution into the sum over l in (3.3) comes from large values l corresponding to $l \lesssim r_H/(\xi - a)$. To the leading order we can replace the summation over l by the integration in accordance with the formula

$$\sum_{l=0}^\infty D_l f(\lambda_l) \to \frac{2r_H^{D-1}}{\Gamma(D-1)}\int_0^\infty dk\, k^{D-2} f(\sqrt{k^2+m^2}). \tag{3.9}$$

Now it is easily seen that from (3.3) we obtain the corresponding result for the plate uniformly accelerated through the Fulling-Rindler vacuum.

In the geometry of two branes, extracting the contribution from the second brane, we can write the expression (3.1) for the VEV in the symmetric form

$$\langle 0\,|\varphi^2|\,0\rangle = \langle 0_0\,|\varphi^2|\,0_0\rangle + \sum_{j=a,b}\langle\varphi^2\rangle^{(j)} + \langle\varphi^2\rangle^{(ab)}, \tag{3.10}$$

with the interference part

$$\langle\varphi^2\rangle^{(ab)} = -\frac{r_H^{1-D}}{\pi S_D}\sum_{l=0}^\infty D_l \int_0^\infty d\omega \bar{I}_\omega^{(a)}(\lambda_l a)\left[\frac{Z_\omega^{(b)2}(\lambda_l\xi,\lambda_l b)}{\bar{I}_\omega^{(b)}(\lambda_l b) Z_\omega(\lambda_l a,\lambda_l b)} - \frac{K_\omega^2(\lambda_l\xi)}{\bar{K}_\omega^{(a)}(\lambda_l a)}\right]. \tag{3.11}$$

An equivalent form for this part is obtained with the replacements (2.31) in the integrand. The interference term (3.11) is finite for all values of ξ in the range $a \leqslant \xi \leqslant b$, including the points on the branes. The surface divergences are contained in the single brane parts only.

Let us consider the behavior of the interference part in the VEV of the field square in limiting regions for values of the parameters. First of all, it can be seen that in the limit $a \to b$, to the leading order the result for the parallel plates in the Minkowski bulk is obtained. When the left brane tends to the horizon, $a \to 0$, the dominant contribution comes from the lower limit of the integration in (3.11), and we have

$$\langle\varphi^2\rangle^{(ab)} \approx \frac{r_H^{1-D}\delta_{B_a}}{2\pi S_D \ln^2(r_H/a)}\sum_{l=0}^\infty D_l \frac{\bar{K}_0^{(b)}(\lambda_l b)}{\bar{I}_0^{(b)}(\lambda_l b)}\left[2K_0(\lambda_l\xi) - \frac{\bar{K}_0^{(b)}(\lambda_l b)}{\bar{I}_0^{(b)}(\lambda_l b)} I_0(\lambda_l\xi)\right] I_0(\lambda_l\xi). \tag{3.12}$$

In the limit $b \to \infty$ for fixed values a and ξ, the main contribution comes from the lowest mode $l = 0$, and to the leading order one finds

$$\langle\varphi^2\rangle^{(ab)} \approx \frac{e^{-2\lambda_0 b}}{S_D r_H^{D-1}}\frac{A_b - B_b\lambda_0}{A_b + B_b\lambda_0}\int_0^\infty d\omega \frac{\bar{I}_\omega^{(a)}(\lambda_0 a)}{\bar{K}_\omega^{(a)}(\lambda_0 a)}\left[2I_\omega(\lambda_0\xi) - \frac{\bar{I}_\omega^{(a)}(\lambda_0 a)}{\bar{K}_\omega^{(a)}(\lambda_0 a)} K_\omega(\lambda_0\xi)\right] K_\omega(\lambda_0\xi), \tag{3.13}$$

with the exponentially suppressed interference part. The behavior of the interference part in the limit $r_H \to 0$ can be investigated in the way similar to that for a single brane part. For a non-minimally coupled scalar field the interference part is dominated by the $l = 0$ term and is suppressed by the factor $\exp[-2\sqrt{\zeta n(n+1)}(b-a)/r_H]$. For a minimally coupled scalar field the leading term is given by the $l = 0$ summand with $\lambda_0 = m$ and the interference part behaves as r_H^{1-D}. In the near-horizon limit, $a, b \ll r_H$, replacing the summation by the integration in accordance with formula (3.9), it can be seen that from (3.11) the result for the geometry of two parallel plates uniformly accelerated through the Fulling-Rindler vacuum is obtained.

3.2 Energy-momentum tensor

The VEV of the energy-momentum tensor is expressed in terms of the Wightman function as

$$\langle 0|T_{ik}|0\rangle = \lim_{x'\to x} \partial_i \partial'_k G^+(x,x') + \left[\left(\zeta - \frac{1}{4}\right) g_{ik}\nabla_l \nabla^l - \zeta \nabla_i \nabla_k - \zeta R_{ik}\right] \langle 0|\varphi^2|0\rangle, \quad (3.14)$$

where R_{ik} is the Ricci tensor for the bulk geometry. Making use of the formulae for the Wightman function and the VEV of the field square, one obtains two equivalent forms, corresponding to $j = a$ and $j = b$ (no summation over i):

$$\begin{aligned}\langle 0|T_i^k|0\rangle &= \langle 0_0|T_i^k|0_0\rangle + \langle T_i^k\rangle^{(j)} - \delta_i^k \frac{r_H^{1-D}}{\pi S_D} \sum_{l=0}^{\infty} D_l \lambda_l^2 \\ &\times \int_0^\infty d\omega\, \Omega_{j\omega}(\lambda_l a, \lambda_l b) F^{(i)}\left[Z_\omega^{(j)}(\lambda_l \xi, \lambda_l j)\right]. \end{aligned} \quad (3.15)$$

In this formula, for a given function $g(z)$ we use the notations

$$F^{(0)}[g(z)] = \left(\frac{1}{2} - 2\zeta\right)\left[\left(\frac{dg(z)}{dz}\right)^2 + \left(1 + \frac{\omega^2}{z^2}\right) g^2(z)\right] + \frac{\zeta}{z}\frac{d}{dz} g^2(z) - \frac{\omega^2}{z^2} g^2(z), \quad (3.16)$$

$$F^{(1)}[g(z)] = -\frac{1}{2}\left(\frac{dg(z)}{dz}\right)^2 - \frac{\zeta}{z}\frac{d}{dz} g^2(z) + \frac{1}{2}\left(1 + \frac{\omega^2}{z^2}\right) g^2(z), \quad (3.17)$$

$$F^{(i)}[g(z)] = \left(\frac{1}{2} - 2\zeta\right)\left[\left(\frac{dg(z)}{dz}\right)^2 + \left(1 + \frac{\omega^2}{z^2}\right) g^2(z)\right] - \frac{g^2(z)}{D-1}\frac{\lambda_l^2 - m^2}{\lambda_l^2}, \quad (3.18)$$

with $g(z) = Z_\omega^{(j)}(z, \lambda_l j)$, where $i = 2, \ldots, D$ and the indices 0,1 correspond to the coordinates τ, ξ, respectively. In formula (3.15),

$$\langle 0_0|T_i^k|0_0\rangle = \delta_i^k \frac{r_H^{1-D}}{\pi^2 S_D} \sum_{l=0}^{\infty} D_l \lambda_l^2 \int_0^\infty d\omega \sinh \pi\omega\, f^{(i)}[K_{i\omega}(\lambda_l \xi)] \quad (3.19)$$

is the corresponding VEV for the vacuum without boundaries, and the term $\langle T_i^k\rangle^{(j)}$ is induced by the presence of a single spherical brane located at $\xi = j$. For the brane at $\xi = a$ and in the region $\xi > a$ one has (no summation over i)

$$\langle T_i^k\rangle^{(a)} = -\delta_i^k \frac{r_H^{1-D}}{\pi S_D} \sum_{l=0}^{\infty} D_l \lambda_l^2 \int_0^\infty d\omega \frac{\bar{I}_\omega^{(a)}(\lambda_l a)}{\bar{K}_\omega^{(a)}(\lambda_l a)} F^{(i)}[K_\omega(\lambda_l \xi)]. \quad (3.20)$$

For the geometry of a single brane at $\xi = b$, the corresponding expression in the region $\xi < b$ is obtained from (3.20) by the replacements (2.31). The expressions for the functions $f^{(i)}[g(z)]$ in (3.19) are obtained from the corresponding expressions for $F^{(i)}[g(z)]$ by the replacement $\omega \to i\omega$. It can be easily seen that for a conformally coupled massless scalar the boundary induced part in the energy-momentum tensor is traceless. The boundary-free part (3.19) and the single brane part $\langle T_i^k \rangle^{(j)}$ in the region $\xi < j$ are investigated in ref. [8].

Now we turn to the investigation of the brane-induced VEVs in limiting cases. First of all let us consider single brane part (3.20). At large distances from the brane, $\xi \gg r_H$, the main contribution comes from the $l = 0$ term and one has

$$\langle T_i^k \rangle^{(a)} \approx \lambda_0^2 \delta_i^k F_0^{(i)} \langle \varphi^2 \rangle^{(a)}, \tag{3.21}$$

where $\langle \varphi^2 \rangle^{(a)}$ is given by (3.5) and

$$F_0^{(0)} = 1 - 4\zeta,\ F_0^{(1)} = \frac{4\zeta - 1}{2\lambda_0 \xi},\ F_0^{(2)} = 1 - 4\zeta - \frac{1 - m^2/\lambda_0^2}{D - 1}. \tag{3.22}$$

In this limit the radial vacuum stress is suppressed by the factor $\lambda_0 \xi$ with respect to the corresponding energy density and azimuthal stresses. In the limit $a \to 0$ when ξ is fixed the main contribution into the ω-integral comes from the lower limit and to the leading order we obtain

$$\langle T_i^k \rangle^{(a)} \approx -\frac{\delta_i^k r_H^{1-D} \delta_{B_a}}{2\pi S_D \ln^2(r_H/a)} \sum_{l=0}^{\infty} D_l \lambda_l^2 F^{(i)} [K_\omega(\lambda_l \xi)]_{\omega=0}. \tag{3.23}$$

For $r_H \to 0$, in the way similar to that for the field square it can be seen that for a non-minimally coupled scalar field $\langle T_i^k \rangle^{(a)}$ is suppressed by the factor $\exp[-2\sqrt{\zeta n(n+1)}(\xi - a)/r_H]$. For a minimally coupled scalar the main contribution comes from the $l = 0$ term and the brane induced VEV (3.20) behaves like r_H^{1-D}.

Now let us present the VEV (3.15) in the form

$$\langle 0 | T_i^k | 0 \rangle = \langle 0_0 | T_i^k | 0_0 \rangle + \sum_{j=a,b} \langle T_i^k \rangle^{(j)} + \langle T_i^k \rangle^{(ab)}, \tag{3.24}$$

where the interference part is given by the formula (no summation over i)

$$\begin{aligned} \langle T_i^k \rangle^{(ab)} &= -\delta_i^k \frac{r_H^{1-D}}{\pi S_D} \sum_{l=0}^{\infty} D_l \lambda_l^2 \int_0^\infty d\omega \bar{I}_\omega^{(a)}(\lambda_l a) \\ &\times \left[\frac{F^{(i)}[Z_\omega^{(b)}(\lambda_l \xi, \lambda_l b)]}{\bar{I}_\omega^{(b)}(\lambda_l b) Z_\omega(\lambda_l a, \lambda_l b)} - \frac{F^{(i)}[K_\omega(\lambda_l \xi)]}{\bar{K}_\omega^{(a)}(\lambda_l a)} \right]. \end{aligned} \tag{3.25}$$

The surface divergences are contained in the single brane parts and the term (3.25) is finite for all values $a \leqslant \xi \leqslant b$. An equivalent formula for $\langle T_i^k \rangle^{(ab)}$ is obtained from eq. (3.25) by replacements (2.31). The behavior of the interference part (3.25) in the limits $a \to 0$ and $b \to \infty$ is similar to that for the field square. In the near-horizon limit, $a, b \ll r_H$, for both single brane and interference parts replacing the summation by the integration in accordance with formula (through the Fulling-Rindler vacuum is obtained.

4. Vacuum interaction forces between the branes

In this section we will consider the vacuum forces acting on the branes. The force acting per unit surface of the brane at $\xi = j$ is determined by the radial component of the vacuum energy-momentum tensor evaluated at this point. By using the decomposition of the VEV for the energy-momentum tensor given by (3.15), the corresponding effective pressures, $p^{(j)} = -\langle T_1^1 \rangle_{\xi=j}$, can be presented as the sum

$$p^{(j)} = p_1^{(j)} + p_{(\text{int})}^{(j)}, \quad j = a, b, \tag{4.1}$$

where the first term on the right is the pressure for a single brane at $\xi = j$ when the second brane is absent. This term is divergent due to the surface divergences in the subtracted vacuum expectation values and needs additional renormalization. This can be done, for example, by applying the generalized zeta function technique to the corresponding mode sum. This procedure is similar to that used in ref. [9] for the evaluation of the surface energy for a single brane. The second term on the right of eq. (4.1), $p_{(\text{int})}^{(j)}$, is the pressure induced by the presence of the second brane, and can be termed as an interaction force. This term determines the force by which the scalar vacuum acts on the brane due to the modification of the spectrum for the zero-point fluctuations by the presence of the second brane. It is finite for all nonzero distances between the branes and is not affected by the renormalization procedure.

For the brane at $\xi = j$ the interaction term is due to the third summand on the right of eq. (3.15). Substituting into this term $i = k = 1$, $\xi = j$ and using the Wronskian relation for the modified Bessel functions one finds

$$p_{(\text{int})}^{(j)} = \frac{A_j^2}{2j^2} \frac{r_H^{1-D}}{\pi S_D} \sum_{l=0}^{\infty} D_l \int_0^{\infty} d\omega \left[\left(\lambda_l^2 j^2 + \omega^2 \right) \beta_j^2 + 4\zeta \beta_j - 1 \right] \Omega_{j\omega}(\lambda_l a, \lambda_l b), \tag{4.2}$$

with $\beta_j = B_j / jA_j$. The interaction force acts on the surface $\xi = a + 0$ for the brane at $\xi = a$ and on the surface $\xi = b - 0$ for the brane at $\xi = b$. In dependence of the values for the coefficients in the boundary conditions, the effective pressures (4.2) can be either positive or negative, leading to repulsive or attractive forces, respectively. For Dirichlet or Neumann boundary conditions on both branes the interaction forces are always attractive. For Dirichlet boundary condition on one brane and Neumann boundary condition on the other one has $p_{(\text{int})}^{(j)} > 0$ and the interaction forces are repulsive for all distances between the branes. Note that the interaction forces can also be written in another equivalent form

$$p_{(\text{int})}^{(j)} = \frac{n^{(j)}}{2j} \frac{r_H^{1-D}}{\pi S_D} \sum_{l=0}^{\infty} D_l \int_0^{\infty} d\omega \left[1 + \frac{(4\zeta - 1)\beta_j}{\left(\lambda_l^2 j^2 + \omega^2 \right) \beta_j^2 + \beta_j - 1} \right] \times \frac{\partial}{\partial j} \ln \left| 1 - \frac{\bar{I}_\omega^{(a)}(\lambda_l a) \bar{K}_\omega^{(b)}(\lambda_l b)}{\bar{I}_\omega^{(b)}(\lambda_l b) \bar{K}_\omega^{(a)}(\lambda_l a)} \right|. \tag{4.3}$$

Now we turn to the investigation of the interaction forces in the asymptotic regions of the parameters. In the limit $a \to b$ the dominant contribution into the expression on the right of (4.2) comes from large values l and ω. Replacing the summation over l by the integration in accordance with $\sum_{l=0}^{\infty} D_l f(l) \to 2\int_0^\infty dl\, l^{D-2} f(l)/\Gamma(D-1)$, and using the uniform asymptotic expansions for the modified Bessel functions, to the leading order we find

$$p_{(\text{int})}^{(j)} \approx \sigma_{ab} \frac{\Gamma\left(\frac{D+1}{2}\right)\zeta_{\text{R}}(D+1)}{(4\pi)^{(D+1)/2}(b-a)^{D+1}}, \tag{4.4}$$

where $\zeta_{\text{R}}(x)$ is the Riemann zeta function, $\sigma_{ab} = -1$ for $\delta_{B_a}\delta_{B_b} = 1$ and $\sigma_{ab} = 1 - 2^{-D}$ for $\delta_{B_a}\delta_{B_b} = -1$. Hence, for small distances between the branes the interaction forces are repulsive for the Dirichlet boundary condition on one brane and non-Dirichlet boundary condition on the other and are attractive for all other cases. Note that in the limit $a \to b$ the interaction part of the total vacuum force acting on the brane diverges, whereas the renormalized single brane parts remain finite. From here it follows that at small distances between the branes the interaction part dominates.

When the left brane tends to the horizon, $a \to 0$, the main contribution into the vacuum interaction forces comes from the lower limit of the ω-integral and one has

$$p_{(\text{int})}^{(a)} \approx -\frac{\delta_{B_a} r_H^{1-D}}{2\pi S_D a^2 \ln^3(r_H/a)} \sum_{l=0}^{\infty} D_l \frac{\bar{K}_0^{(b)}(\lambda_l b)}{\bar{I}_0^{(b)}(\lambda_l b)}, \tag{4.5}$$

$$p_{(\text{int})}^{(b)} \approx \frac{r_H^{1-D}\delta_{B_a} A_b^2}{4\pi S_D b^2 \ln^2(r_H/a)} \sum_{l=0}^{\infty} D_l \frac{\lambda_l^2 b^2 \beta_b^2 + 4\zeta\beta_b - 1}{\bar{I}_0^{(b)2}(\lambda_l b)}. \tag{4.6}$$

In this limit the interaction forces have different signs for the Dirichlet and non-Dirichlet boundary conditions on the brane $\xi = a$. The combination $\delta_{B_a} p_{(\text{int})}^{(j)}$ is positive for large values β_b and is negative for small values of this parameter. In the limit $b \to \infty$ for fixed a, the dominant contribution comes from the lowest mode $l = 0$ and assuming that $A_b \neq \pm\lambda_0 B_b$, we have the estimates

$$p_{(\text{int})}^{(a)} \approx \frac{A_a^2 e^{-2\lambda_0 b}}{2S_D a^2 r_H^{D-1}} \frac{A_b - \lambda_0 B_b}{A_b + \lambda_0 B_b} \int_0^\infty d\omega\, \frac{\left(\lambda_l^2 a^2 + \omega^2\right)\beta_a^2 + 4\zeta\beta_a - 1}{\bar{K}_0^{(a)2}(\lambda_0 a)}, \tag{4.7}$$

$$p_{(\text{int})}^{(b)} \approx -\frac{\lambda_0 e^{-2\lambda_0 b}}{S_D b r_H^{D-1}} \frac{A_b - \lambda_0 B_b}{A_b + \lambda_0 B_b} \int_0^\infty d\omega \frac{\bar{I}_\omega^{(a)}(\lambda_0 a)}{\bar{K}_\omega^{(a)}(\lambda_0 a)}, \tag{4.8}$$

with the exponentially small interaction forces for both branes. In particular, the combination $(A_b^2 - \lambda_0^2 B_b^2) p_{(\text{int})}^{(j)}$ is positive/negative for large/small values β_a. For small values of the curvature radius r_H, in the way similar to that used for the VEV of the field square, it can be seen that for a non-minimally coupled scalar field the main contribution comes from the $l = 0$ term and to the leading order we have

$$p_{(\text{int})}^{(j)} \approx -\frac{\delta_{B_a}\delta_{B_b}[\zeta n(n+1)]^{3/4}\sqrt{ab}}{2\sqrt{\pi} S_D r_H^{D+1/2} j\sqrt{b-a}} \exp[-2\sqrt{\zeta n(n+1)}(b-a)/r_H]. \tag{4.9}$$

For a minimally coupled scalar field the dominant contribution comes from the $l = 0$ term with $\lambda_0 = m$ and the interaction forces per unit surface behave like r_H^{1-D}. In the near-horizon limit, $a, b \ll r_H$, in (4.2) we replace the summation over l by the integration in accordance with (3.9) and to the leading order the result for the geometry of two parallel plates uniformly accelerated through the Fulling-Rindler vacuum is obtained.

5. Conclusion

In this paper, we investigate the polarization of the scalar vacuum induced by two spherical branes in the $(D+1)$-dimensional bulk $Ri \times S^{D-1}$, assuming that on the branes the field obeys the Robin boundary conditions. In the corresponding braneworld scenario based on the orbifolded version of the model the coefficients in the boundary conditions are expressed in terms of the brane mass parameters by formula (2.6). The most important characteristics of the vacuum properties are the expectation values of quantities bilinear in the field operator such as the field square and the energy-momentum tensor. As the first step in the investigation of these VEVs we evaluate the positive frequency Wightman function. The corresponding mode sum contains the summation over the eigenfrequencies. In the region between the branes the latter are the zeros of the bilinear combination of the modified Bessel functions and their derivatives. For the summation of the series over these zeros we employ a variant of the generalized Abel-Plana formula. This allows us to present the Wightman function as the sum of a single brane and second brane induced parts, formulae (2.21) and (2.27).

The corresponding VEVs of the field square and the energy-momentum tensor are obtained from the Wightman function in the coincidence limit and are investigated in section 3. These VEVs are given by formulae (representations. We have considered various limiting cases of the general formulae. In particular, we have shown that when the left brane tends to the horizon the interference parts in the VEVs of the field square and the energy-momentum tensor vanish as $1/\ln^2(r_H/a)$. In the limit when the right brane tends to infinity, $b \to \infty$, the interference parts are suppressed by the factor $\exp(-2\lambda_0 b)$. In the high curvature regime, corresponding to small values r_H, the behavior of the VEVs is essentially different for minimally and non-minimally coupled scalar fields. For a non-minimally coupled field the VEVs are suppressed by the factor $\exp[-2\sqrt{\zeta n(n+1)}(\xi - a)/r_H]$ for single brane parts and by $\exp[-2\sqrt{\zeta n(n+1)}(b-a)/r_H]$ for the interference parts. For a minimally coupled field the main contribution comes from the $l = 0$ term and the VEVs behave as r_H^{1-D}. In the limit when the both branes are near the horizon, to the leading order the VEVs are obtained for the geometry of two parallel plates uniformly accelerated through the Fulling-Rindler vacuum.

In section 4 we have investigated the vacuum forces acting on the branes. These forces are presented as the sum of self-action and interaction parts. Due to the well-known surface divergences, the self-action part needs additional subtractions. The interaction forces are finite for all nonzero interbrane distances and are given by formula (4.2) or equivalently by (4.3). In general, these forces are different for the left and right branes and, in dependence of the values of the parameters, they can be either attractive or repulsive. In particular, at small interbrane distances they are repulsive for the Dirichlet boundary condition on one brane and non-Dirichlet boundary condition on the other, and are attractive for all other cases. When the left brane tends to the horizon the interaction forces acting on the left and right branes behave as $1/[a^2 \ln(r_H/a)]$ and $1/\ln^2(r_H/a)$, respectively. In the limit when the ξ-coordinate of the right brane tends to infinity, the interaction forces for both branes are suppressed by the factor $\exp(-2\lambda_0 b)$.

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